THERMAL ANALYSIS OF A PERMEABLE WALL FOR GIVEN THERMAL FLUXES ON ITS SURFACE

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An analysis is presented of the tempeature field for a permeable (porous or perforated) wall for a plate, cylinder, and sphere under boundary conditions of the second kind.

Systems consisting of a permeable wall through which an injectant (liquid or gas) is filtered are used extensively in the machine construction and chemical industries as well as in energetics. Despite the urgency of the problem of an analytical analysis of the internal heat and mass transfer in permeable media, this question is studied inadequately in the technical literature [1-6]. The author obtained a solution of this problem in [1-4] for boundary conditions of the first and third kinds. Design dependences for boundary conditions of the second kind are presented below for the following formulation of the problem.

An injectant with the initial temperature T_ε and density j_i of the transverse flux of material is filtered through a wall with porosity Π from the "cold" to the hot surface in the presence of internal energy sources or sinks with the specific power $q_V(y)$. The temperatures on the "cold" ($y = y_1$) and the "hot" surface for $y = y_2$ are T_1 and T_2, respectively. The thermal flux density at $y = y_2$ is $Q_{2\Gamma}$. It is necessary to find the temperature field in the permeable wall (plane, cylindrical, or spherical) under phase transformation conditions on the "hot" surface of a hood with the influence of all the fundamental process parameters taken into account.

The differential equations for the temperatures of the permeable wall and the injectant, whose derivation and foundation are presented in [1, 6] in the dimensionless variables $t = T/T_\infty$ and $\bar{y} = y/y_2$, have the form (here and henceforth the prime denotes the derivative with respect to $\bar{y}$)*

$$\bar{y}^{\Gamma} t_{\Gamma}'' + (\Gamma\bar{y}^{\Gamma-1} - \xi_{\Gamma}) t_{\Gamma}' + Q_{\Gamma} = 0, \tag{1}$$

$$\bar{y}^{\Gamma} t_{\Gamma i}'' + (\Gamma\bar{y}^{\Gamma-1} - \xi_{\Gamma i}) t_{\Gamma i}' = 0, \quad \Gamma = p, c, s, \tag{2}$$

under the following boundary conditions: $\bar{y} = -\infty$ (plate); $\bar{y} = 0$ (cylinder and sphere),

$$t_i = t_\varepsilon, \tag{3}$$

$$\bar{y} = \bar{y}_1, \quad t_{\Gamma} = t_{\Gamma 1}, \tag{4}$$

$$\lambda_{i\Sigma} t_i'(\bar{y}_1) = q_{1\Gamma}, \tag{5}$$

$$\bar{y} = 1, \quad t_{\Gamma} = t_{\Gamma 2}, \tag{6}$$

$$L_{\Gamma} + q_{2\Gamma} = \bar{q}_{2\Gamma}. \tag{7}$$

Here

$$q_{1\Gamma} = t_{\Gamma}'(\bar{y}_1), \quad q_{2\Gamma} = t_{\Gamma}'(1),$$

*The subscript Γ indicates what body (one of the three being investigated) is considered, Γ = p, c, s. If Γ is a factor or exponent, then it will be 0.1 and 2, respectively, for the plate, cylinder, and sphere, where $2\pi\Gamma = 1$ for the plate.

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$$\lambda_{i\Sigma}=\frac{\lambda_i}{\lambda_\Sigma},\qquad L_\Gamma=\frac{rj_\Gamma y_2^{1-\Gamma}}{\lambda_\Sigma T_\infty},\qquad Q_\Gamma=\frac{q_V(\bar y)\,\bar y^\Gamma}{\lambda_\Sigma T_\infty y_2^{-2}},$$

$$\bar q_{2\Gamma}=\frac{Q_{2\Gamma}y_2}{\lambda_\Sigma T_\infty},\qquad \xi_\Gamma=\frac{j_\Gamma c_{pi}}{\lambda_\Sigma y_2^{\Gamma-1}},\qquad \xi_{\Gamma i}=\frac{j_\Gamma c_{pi}}{\lambda_i y_2^{\Gamma-1}},$$

and $j_\Gamma = j_i(\bar y_1)y_1^\Gamma$, r is the heat of the phase transformations.

Condition (5) characterizes the equality of the conductive thermal fluxes from the injectant and the "cold" wall surface. According to condition (7), the thermal flux $\bar q_{2\Gamma}$ delivered to the permeable wall is expended in the phase transformations and heating of the body skeleton.

Let us first find the solution to the problem (2)-(4). It has the form

$$\begin{aligned}t_{pi}&=(t_1-t_\varepsilon)\exp\xi_{pi}(\bar y-\bar y_1)+t_\varepsilon,\quad -\infty\leqslant\bar y\leqslant\bar y_1,\\ t_{ci}&=(t_1-t_\varepsilon)\left(\frac{\bar y}{\bar y_1}\right)^{\xi_{ci}}+t_\varepsilon,\quad 0\leqslant\bar y\leqslant\bar y_1,\\ t_{si}&=(t_1-t_\varepsilon)\exp\xi_{si}\left(\frac{1}{\bar y_1}-\frac{1}{\bar y}\right)+t_\varepsilon,\quad 0\leqslant\bar y\leqslant\bar y_1.\end{aligned}\tag{8}$$

The solution of the differential equation for the permeable body temperature (1) under the boundary conditions (4) and (7) is obtained in two quadratures:

$$t_\Gamma=t_{1\Gamma}+\frac{1}{\xi_\Gamma}\int_{\bar y_1}^{\bar y}Q_\Gamma(\bar y)\,d\bar y-\frac{Z_\Gamma(\bar y)}{\xi_\Gamma}+\frac{1}{\xi_\Gamma}[\bar q_{2\Gamma}-L_\Gamma+Z_\Gamma(1)][\psi_\Gamma(\bar y)-\psi_\Gamma(\bar y_1)],\quad \Gamma=p,c,s,\tag{9}$$

where

$$\begin{aligned}Z_p(\bar y)&=\exp\xi_p\bar y\int_{\bar y_1}^{\bar y}Q_p(\bar y)\exp(-\xi_p\bar y)\,d\bar y,\\ Z_c(\bar y)&=\bar y^{\xi_c}\int_{\bar y_1}^{\bar y}\bar y^{-\xi_c}Q_c(\bar y)\,d\bar y,\\ Z_s(\bar y)&=\exp(-\xi_s\bar y)\int_{\bar y_1}^{\bar y}Q_s(\bar y)\exp\left(\frac{\xi_s}{\bar y}\right)d\bar y,\end{aligned}\tag{10}$$

$$\psi_p(\bar y)=\exp\xi_p(\bar y-1),\quad \psi_c(\bar y)=\bar y^{\xi_c},\quad \psi_s(\bar y)=\exp\xi_s\left(1-\frac{1}{\bar y}\right),\tag{11}$$

$$Z_\Gamma(1)=Z_\Gamma(\bar y)|_{\bar y=1},\quad \psi_\Gamma(\bar y_1)=\psi_\Gamma(\bar y)|_{\bar y=\bar y_1}.$$

For $Q_\Gamma = \gamma_\Gamma$ = const formulas (9) transform into the simpler form

$$t_\Gamma=t_{1\Gamma}+\xi_\Gamma^{-1}\gamma_\Gamma(\bar y-\bar y_1)-\xi_\Gamma^{-1}\bar Z_\Gamma(\bar y)+\xi_\Gamma^{-1}[\bar q_2-L_\Gamma+\bar Z_\Gamma(1)][\psi_\Gamma(\bar y)-\psi_\Gamma(\bar y_1)].\tag{12}$$

Here

$$\begin{aligned}\bar Z_p(\bar y)&=\xi_p^{-1}\gamma_p[\exp\xi_p(\bar y-\bar y_1)-1],\\ \bar Z_c(\bar y)&=\frac{\gamma_c\bar y^{\xi_c}}{1-\xi_c}(\bar y^{1-\xi_c}-\bar y_1^{1-\xi_c}),\\ \bar Z_s(\bar y)&=\gamma_s И(\bar y,\bar y_1)\exp(-\xi_s\bar y),\end{aligned}\tag{13}$$

where

$$И(\bar y,\bar y_1)=\int_{\bar y_1}^{\bar y}\exp\frac{\xi_s}{\bar y}\,d\bar y.$$

Differentiating the solutions (8) and (9) in conformity with the boundary condition (5), we find the value of the temperature $t_{1\Gamma}$ on the wall "cold" surface which enters into (9):

TABLE 1. Influence of the Dimensionless Blowing Parameter ξ_Γ on the Distribution of the Temperature t_Γ over the Porous Wall Thickness $\bar{y}$ and Values of $q_{1\Gamma}$ for $q_{2\Gamma}$ = 0.999

Γ	ξ_Γ	$q_{1\Gamma}$	$\bar{y}$ = 0,80	0,82	0,84	0,86	0,88	0,90	0,92	0,94	0,96	0,98	1,00
			$t_\Gamma \cdot 10^3$										
	2	1,49	847	876	905	933	960	986	1010	1030	1060	1080	1100
	3	1,30	533	559	585	609	634	657	680	703	725	746	766
0	5	1,00	300	320	340	360	380	400	420	440	460	480	500
	7	0,784	212	228	244	260	277	249	311	329	347	366	386
	10	0,568	157	168	180	192	205	218	232	247	263	281	300
	2	1,80	820	855	888	920	951	980	1010	1030	1060	1080	1100
	3	1,54	510	541	570	598	626	652	677	701	724	746	766
1	5	1,15	284	306	329	351	374	395	417	438	459	480	500
	7	0,887	200	218	236	253	272	290	308	327	346	366	386
	10	0,615	149	162	174	188	201	215	230	246	263	281	300
	2	2,15	789	831	870	907	941	973	1000	1030	1060	1080	1100
	3	1,81	485	521	555	587	617	646	673	699	723	746	766
2	5	1,20	266	292	317	342	367	391	414	436	458	479	500
	7	0,964	188	207	227	247	266	286	306	326	346	366	386
	10	0,655	142	155	169	183	197	212	228	245	262	280	300

TABLE 2. Dependence of t_Γ on $\bar{y}$ for Different Values of the Parameter L_Γ

Γ	L_Γ	$q_{1\Gamma}$	$q_{2\Gamma}$	$\bar{y}$ = 0,80	0,82	0,84	0,86	0,88	0,90	0,92	0,94	0,96	0,98	1,00
				$t_\Gamma \cdot 10^3$										
	0,011	1,13	0,989	383	406	428	450	472	494	515	536	557	577	597
0	0,051	1,11	0,949	379	401	423	445	466	487	508	528	548	568	587
	0,081	1,10	0,919	375	397	419	440	461	482	502	522	542	561	580
	0,011	1,32	0,989	364	390	416	441	465	489	512	534	556	577	597
1	0,051	1,30	0,949	360	386	411	435	459	482	505	527	548	568	587
	0,081	1,28	0,919	357	382	407	431	455	477	499	521	541	561	580
	0,011	1,52	0,989	343	373	402	430	458	484	509	533	555	577	597
2	0,051	1,50	0,949	340	369	398	425	452	477	502	525	547	568	587
	0,081	1,48	0,919	337	366	394	421	447	472	496	519	540	561	580

$$t_{1\Gamma} = t_\varepsilon + p_\Gamma \psi_\Gamma (\bar{y}_1), \quad \Gamma = \text{p, c, s}. \tag{14}$$

For $Q_\Gamma = \gamma_\Gamma$ = const we obtain from (14)

$$t_{1\Gamma} = t_\varepsilon + \bar{p}_\Gamma \psi_\Gamma (\bar{y}_1).$$

Here

$$p_\Gamma = \xi_\Gamma^{-1} [\bar{q}_{2\Gamma} - L_\Gamma + \gamma_\Gamma \varkappa_\Gamma (\bar{y}_1)], \tag{15}$$

where

$$\begin{gathered} \varkappa_p (\bar{y}_1) = \xi_p^{-1} [\exp \xi_p (1 - \bar{y}_1) - 1], \\ \varkappa_c (\bar{y}_1) = \frac{1 - \bar{y}_1^{\xi_c}}{1 - \xi_c}, \\ \varkappa_s (\bar{y}_1) = И (1, \bar{y}_1) \exp (-\xi_s). \end{gathered} \tag{16}$$

The absolute values of the thermal fluxes $q_{1\Gamma}$ and $q_{2\Gamma}$ in the boundary conditions (5) and (7) are found by differentiating the solution (9) with (14) taken into account:

$$q_{1\Gamma} = \bar{y}_1^{-\Gamma} [\bar{q}_{2\Gamma} - L_\Gamma + Z_\Gamma (1)] \psi_\Gamma (\bar{y}_1), \tag{17}$$

$$q_{2\Gamma} = \bar{q}_{2\Gamma} - L_\Gamma, \tag{18}$$

where (18) agrees with (7).

For $Q_\Gamma = \gamma_\Gamma$ = const we obtain from (17)

$$q_{1\Gamma} = \bar{y}_1^{-\Gamma} [\bar{q}_{2\Gamma} - L_\Gamma + \bar{Z}_\Gamma (1)] \psi_\Gamma (\bar{y}_1).$$

The results of computing the dimensionless temperature of the permeable wall t_Γ and the thermal fluxes are presented in Tables 1 and 2, as well as in Figs. 1-3. The data presented

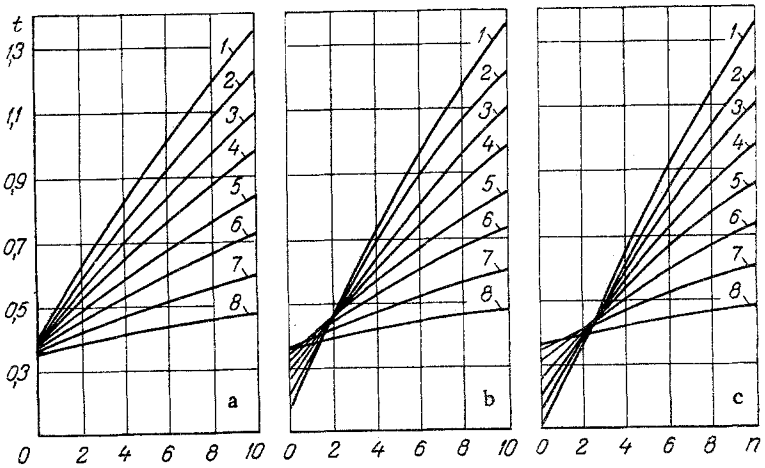

Fig. 1. Distribution of the dimensionless temperature t over the wall thickness $l = 1 - \bar{y}_1$, divided into n parts, for $\bar{y} = 0$ and $\bar{y} = 1$, respectively, n = 0 and n = 10 [1) $\bar{y}_1 = 0.2$; 2) 0.3; 3) 0.4; 4) 0.5; 5) 0.6; 6) 0.7; 7) 0.8; 8) 0.9]. Here and in Figs. 2 and 3: a for a plate; b) for a cylinder; c) for a sphere.

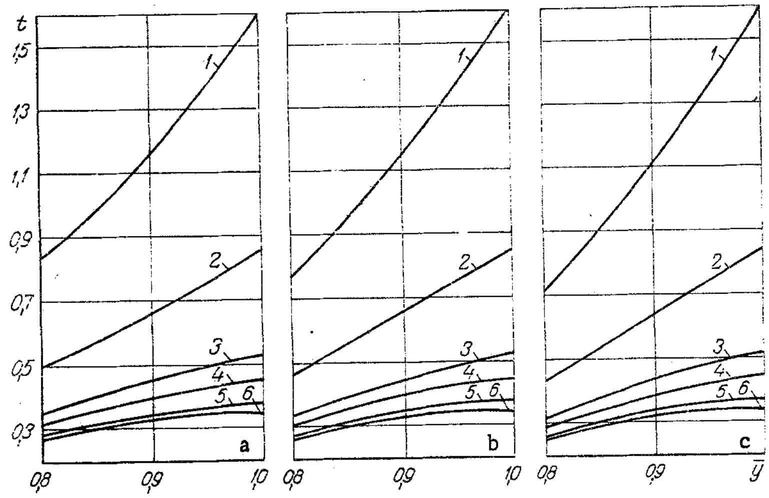

Fig. 2. Distribution of the temperature t over the wall thickness $\bar{y}$ as a function of the dimensionless thermal flux $\bar{q}_{2\Gamma}$ to the "hot" surface: 1) $\bar{q}_{2\Gamma} = 5.0$; 2) 2.0; 3) 0.7; 4) 0.4; 5) 0.1; 6) 0.

have been obtained for $\xi_\Gamma = 4$, $\gamma_\Gamma = 5$, $L_\Gamma = 0.001$, $\bar{q}_{2\Gamma} = 1$. One of these parameters is assumed variable in order to analyze its influence on the process under consideration.

The data obtained confirm the fact that the values of the temperature t_Γ and their gradients over the wall thickness diminish as the dimensionless blowing velocity ξ_Γ increases. The $q_{1\Gamma}$ diminish correspondingly; hence the values of $q_{2\Gamma}$ remain constant since the computation was performed for $\bar{q}_{2\Gamma} = 1$. As the heat expenditure in the phase transformations, characterized by the parameter L_Γ (Table 2), increases, the values of $t_{1\Gamma}$, $q_{1\Gamma}$, and $q_{2\Gamma}$ diminish, which is explained by condition (7). As the wall thickness $l = y_2 - y_1$ increases in proportion to the number n (n = 0 and n = 10, respectively, for $y = y_1$ and $y = y_2$), the values of the dimensionless temperatures t_Γ and the differences $\Delta t = t_2 - t_1$ grow (Fig. 1). As should be expected, the values of t_Γ also rise with the growth of the flux $\bar{q}_{2\Gamma}$ delivered to the wall hot surface (Fig. 2), as well as the energy source intensity γ_Γ (Fig. 3).

The greatest values of the temperature t_Γ and the differences $\Delta t_\Gamma = t_{2\Gamma} - t_{1\Gamma}$ are observed for a sphere and the least for a plate in all the cases considered, which is explained by the following relationship governing the change in the weight discharge of the injectant over the wall thickness of the porous body:

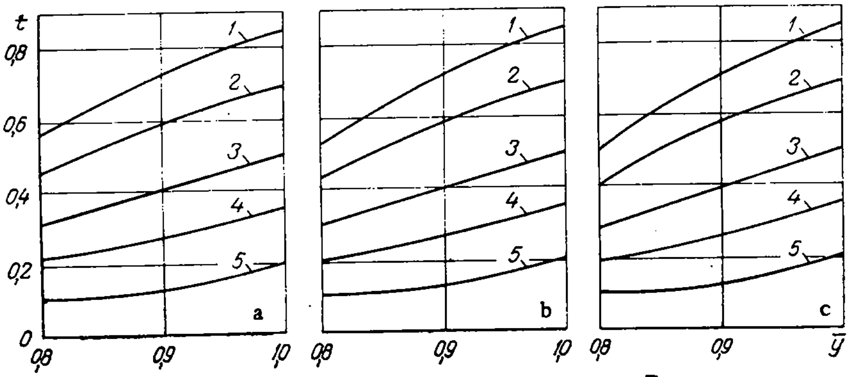

Fig. 3. Dependence of t on the coordinate $\bar{y}$ and internal energy intensity γ_Γ: 1) $\gamma_\Gamma = 10$; 2) 7; 3) 3; 4) 0; 5) −3.

TABLE 3. Analysis of Parameters ξ_Γ, $\bar{q}_{2\Gamma}$ for λ_Σ = 0.418 W/m·deg

Gas being filtered	$j_i \cdot 10^3$, kg/m²·sec	c_{pi}, kJ/kg·deg	T_2, K	ξ_Γ	$\bar{q}_{2\Gamma}$
Natural	5,83	1,25	1070	1,75	4,6
	1,80	1,25	900	0,53	1,1
Coke	3,62	3,56	1000	3,07	8,5
Liquefied	3,33	4,18	1140	4,0	9,7

$$j_i(\bar{y}) = j_i(\bar{y}_1)\left(\frac{y_1}{y}\right)^{\Gamma},$$

where $j_i(\bar{y}_1)$ is the value of j_i at the "cold" surface of the permeable wall.

The dependences (9) and (12) have been obtained under the assumption of constant thermophysical properties. As the analysis performed showed, the error in the analysis because of such an assumption is around 10%, which corresponds to experimental accuracy [6].

Values of the dimensionless parameters ξ_Γ and $\bar{q}_{2\Gamma}$ computed at T_ε = 293°K, y_2 = 0.1 are presented in Table 3 according to test data obtained for a porous ceramic radiator [7].

The values of L_Γ used in the computations correspond to the actual densities of a transverse flux of liquid injectant j_i which evaporates on the body surface. Thus, in the case of water evaporation $j_i = 0.24 \cdot 10^{-4}$ and $j_i = 1.76 \cdot 10^{-4}$ kg/m²·sec, we have L_Γ = 0.011 and L_Γ = 0.081, respectively. Corresponding to the value q_V = 61.25 kW/m³ is γ_Γ = 5. Therefore, the considered values of the dimensionless parameters correspond to the actual range of their variation under practical conditions. In the general case, the solutions obtained are valid for values of L_Γ, Q_Γ, and ξ_Γ which vary between 0 and ∞ [1].

If $\bar{q}_{2\Gamma} = (1 - t_2)\alpha y_2/\lambda_\Sigma$, where α is the heat transfer coefficient, then the solutions under the boundary conditions of the second and third kinds agree [1]. In contrast to the boundary conditions of the first kind, the heat balance equation for $\bar{y}$ = 1 hence permits finding t_2 and taking account of the influence of energy sources or sinks on this surface.

NOTATION

T, permeable wall temperature; Π, its porosity; y, a coordinate normal to the surface of the permeable body; λ, heat conduction coefficient; $\lambda_\Sigma = (1 - \Pi)\lambda_T + \Pi\lambda_L$. Subscripts: T, porous body skeleton; i, injectant; Σ, total (effective) value; 1, "cold" wall surface; 2, "hot" wall surface, ε, values as $y \to -\infty$, ∞ as $y \to +\infty$; p, plate; c, cylinder; and s, sphere.

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OPTIMIZATION OF CONVECTIVE CIRCULAR FINS

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The volume of a circular fin whose thickness is inversely proportional to the square of the radius is optimized.

The books [1, 2] provide an idea of the present state of the theory and practical application of finned heating surfaces. These books also examine the question of optimizing the volume of the fins. The object of optimization is to select a fin with minimum volume for transferring a specified amount of heat under known thermophysical conditions. Whereas for straight fins the problem of optimization is solved by several variants of the cross section of the fin, for circular fins only the results of [3] for fins of constant thickness are given.

We note that for hyperbolic profiles examined in [1, Tables 1-5], the problem of optimization is solved very simply in the case of the thickness of the fin being inversely proportional to the square of the radius. If we use the notation of [1], this dependence has the form

$$\delta/\delta_1 = R^{-2}, \tag{1}$$

$$R = r/r_1. \tag{2}$$

For convenience, we denote the height of the fin

$$h = r_2 - r_1, \tag{3}$$

and the parameter of the fin N is expressed in the form

$$N^2 = 2\alpha h^2/\lambda\delta_1. \tag{4}$$

To make the circular rib more comparable with a straight rib, we refer the thermal flux and the volume of the circular rib to a unit length of the base

$$Q_1 = Q_0/2\pi r_1 = \alpha\vartheta_1\eta h(R_2 + 1), \tag{5}$$

$$V_1 = \frac{1}{2\pi r_1}\int_{r_1}^{r_2} 2\pi r\delta dr = h\delta_1 \ln R_2/(R_2 - 1). \tag{6}$$

Determining the value of δ_1/h^2 from (4), and h from (5), we can express the product in (6) as

$$h\delta_1 = (\delta_1/h^2)(h^3), \tag{7}$$

and formula (6) is transformed to the form

$$V_1 = \left(\frac{Q_1}{\alpha\vartheta_1}\right)^3 \frac{2\alpha}{\lambda N^2} \frac{\ln R_2}{\eta^3(R_2 + 1)^3(R_2 - 1)}. \tag{8}$$

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